Université de Toulouse

IUT Paul Sabatier Département Génie Civil

Semestre 1 – UE13 Sciences et Techniques - Module SST1

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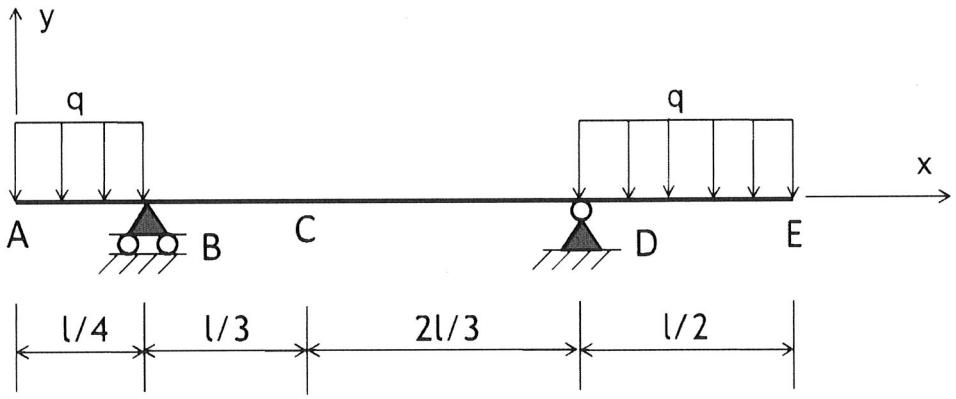
21 Décembre 2018

CONTROLE N°2 DE STRUCTURES ET STABILITE 1 (SST1)

(Durée 2h – sans document, calculatrice collège)

Exercice 1 (9 points)



1.1. Faire le schéma mécanique de la poutre sur la copie.

1.2. Calculer les actions de liaison externe.

1.3. Faire une coupure entre A et B à une abscisse x de A et déterminer les efforts internes dans la section correspondante, en fonction de x.

1.4. Même chose entre B et D.

1.5. Même chose entre D et E.

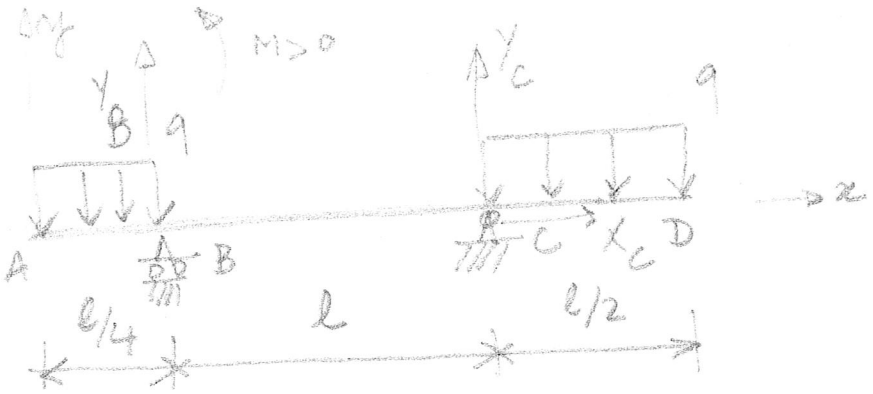
1.6. Déterminer les moments fléchissants en A, B, D et E

1.7. Tracer les diagrammes des efforts internes sur toute la longueur de la poutre (à partir des résultats des questions 1.3, 1.4 et 1.5 ou par une méthode graphique). Indiquer les valeurs particulières sur les diagrammes.

Exercice 2 (6 points)

2.1. Faire le schéma mécanique de la poutre (voir au recto) sur la copie.

2.2. Calculer les actions de liaison externe.



1°) -

$$\begin{cases} X_B = 0 \\ Y_B + Y_C - \frac{q\ell}{4} - \frac{q\ell}{2} = 0 \\ Y_C \times \ell + \frac{q\ell}{4} \times \frac{\ell}{8} - \frac{q\ell}{2} \times \left(\ell + \frac{\ell}{4}\right) = 0 \end{cases}$$

$$Y_C = -\frac{q\ell}{32} + \frac{5q\ell}{8} \Leftrightarrow \boxed{Y_C = \frac{19q\ell}{32}}$$

$$Y_B = \frac{3q\ell}{4} - \frac{19q\ell}{32} \Leftrightarrow \boxed{Y_B = \frac{5q\ell}{32}}$$

2°) - Coupure entre A et B $\quad 0 < x < \ell/4$

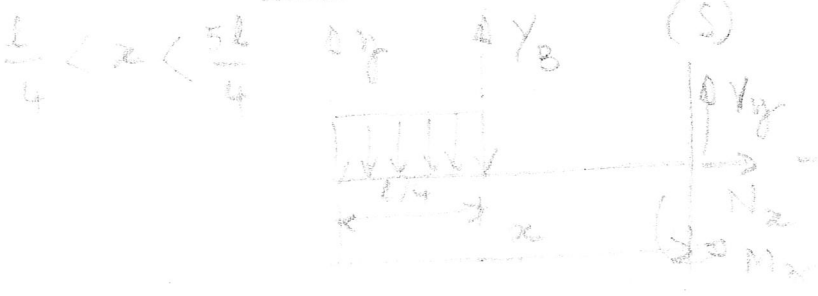
$$V_y - qx = 0$$

$$\boxed{V_{y\,AB}(x) = qx}$$

$$M_z + qx \times \frac{x}{2} = 0$$

$$\boxed{M_{z\,AB}(x) = -\frac{qx^2}{2}}$$

Coupure entre B et C $\quad \frac{\ell}{4} < x < \frac{5\ell}{4}$



$$V_y - \frac{q\ell}{4} + Y_B = 0$$

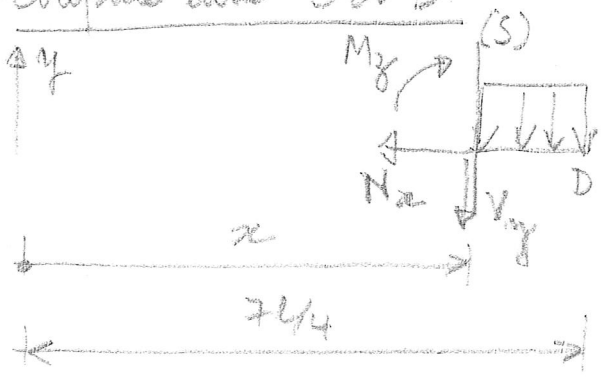
$$\boxed{V_{y\,BC}(x) = \frac{3q\ell}{32}}$$

$$M_{fz} + q\frac{l}{4}x\left(x-\frac{l}{8}\right) - Y_B \times \left(x-\frac{l}{4}\right) = 0$$

$$M_{fz} = -\frac{ql}{4}x + \frac{ql^2}{32} + \frac{5ql}{32}x - \frac{5ql^2}{128}$$

$$\boxed{M_{fzBC}(x) = -\frac{3ql}{32}x - \frac{ql^2}{128}}$$

Coupure entre C et D $\quad \frac{5l}{4} < x < \frac{7l}{4}$



$$-V_{fy} - q\left(\frac{7l}{4} - x\right) = 0$$

$$\boxed{V_{fyCD}(x) = qx - \frac{7ql}{4}}$$

$$-M_{fz}(x) - q\left(\frac{7l}{4} - x\right)\times \frac{1}{2}\left(\frac{7l}{4} - x\right) = 0$$

$$\boxed{M_{fzCD}(x) = -\frac{1}{2}q\left(\frac{7l}{4} - x\right)^2}$$

3°) Valeurs aux bornes

$$V_{fyAB}(0) = 0$$

$$V_{fyAB}\left(\frac{l}{4}\right) = \frac{ql}{4}$$

$$V_{fyBC}\left(\frac{l}{4}\right) = V_{fyBC}\left(\frac{5l}{4}\right) = \frac{3ql}{32}$$

$$V_{fyCD}\left(\frac{5l}{4}\right) = -\frac{ql}{2}$$

$$V_{fyCD}\left(\frac{7l}{4}\right) = 0$$

$$M_{fyAB}(0) = 0$$

$$M_{fyAB}\left(\frac{l}{4}\right) = -\frac{ql^2}{32}$$

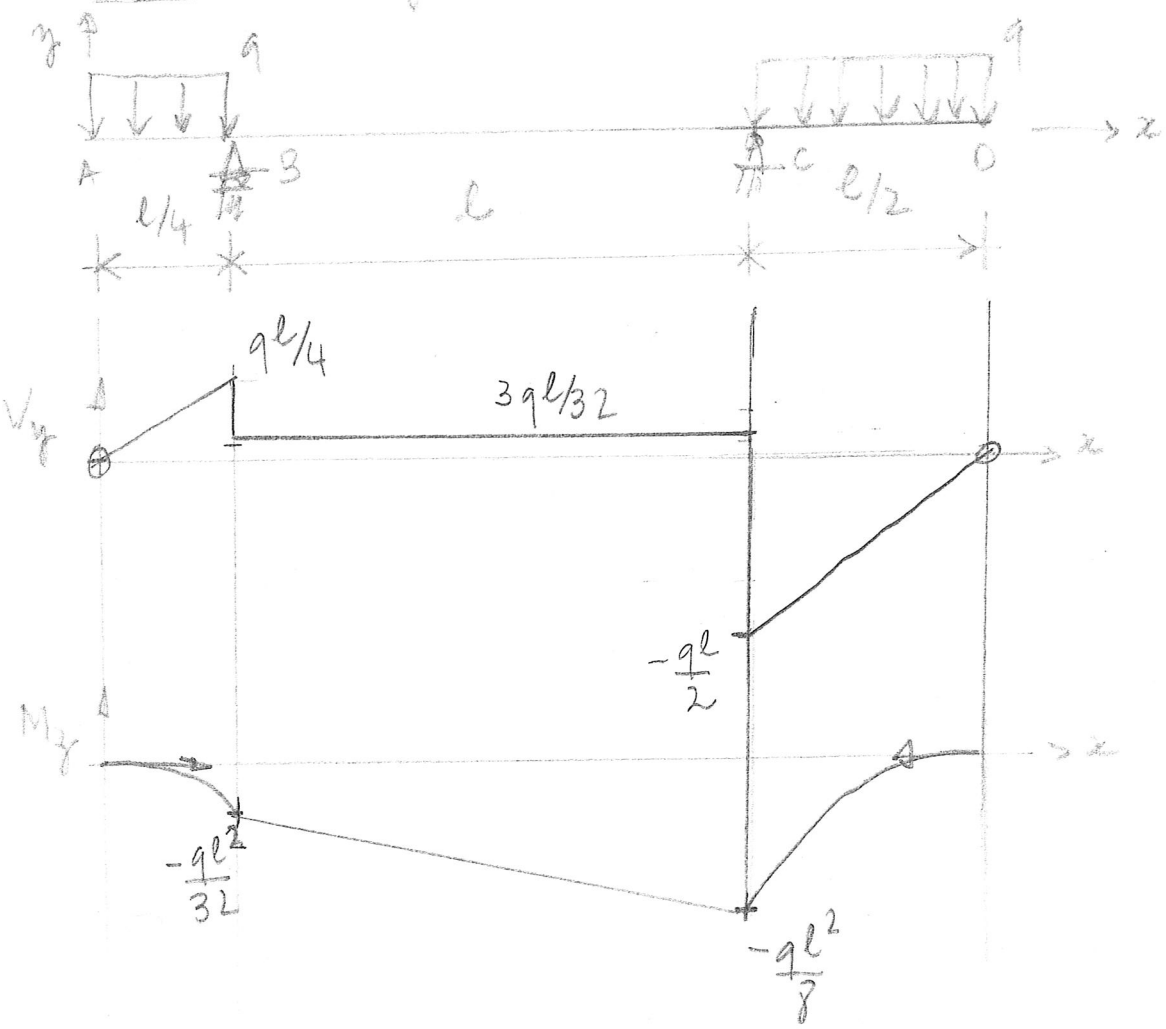
$$M_{fyBC}\left(\frac{l}{4}\right) = -\frac{ql^2}{32}$$

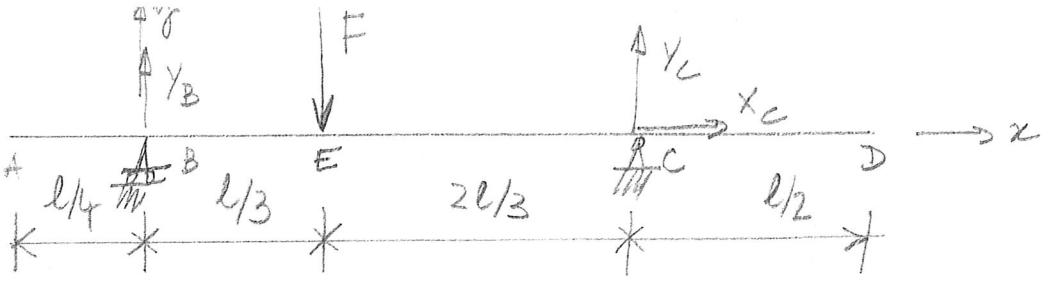
$$M_{fyBC}\left(\frac{5l}{4}\right) = -\frac{ql^2}{8}$$

$$M_{fyCD}\left(\frac{5l}{4}\right) = -\frac{ql^2}{8}$$

$$M_{fyCD}\left(\frac{7l}{4}\right) = 0$$

4°) Tracé des diagrammes :





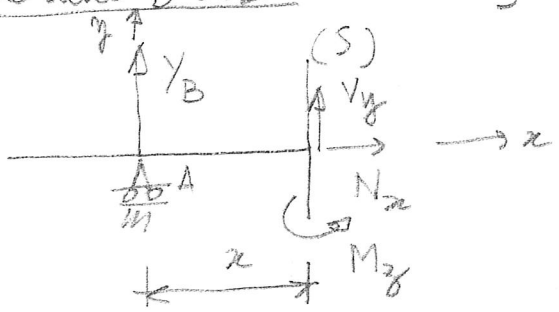
1°) $X_C = 0$

$Y_B + Y_C - F = 0$ $\quad Y_B = F - Y_C \Leftrightarrow \boxed{Y_B = \frac{2F}{3}}$

$Y_C \times l - F \times \frac{l}{3} = 0$ $\quad \boxed{Y_C = \frac{F}{3}}$

2°)

Coupure entre B et E $\quad 0 < x < \frac{l}{3}$



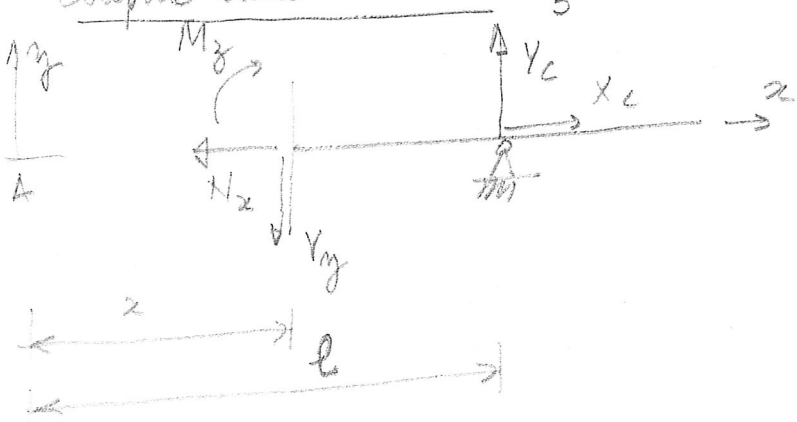
$V_y + Y_B = 0$

$\boxed{V_{y_{BE}} = \frac{-2F}{3}}$

$M_z - Y_B \times x = 0$

$M_{z_{BE}}(x) = \frac{2F}{3} x$

Coupure entre E et C $\quad \frac{l}{3} < x < l$



$-V_y + Y_C = 0$

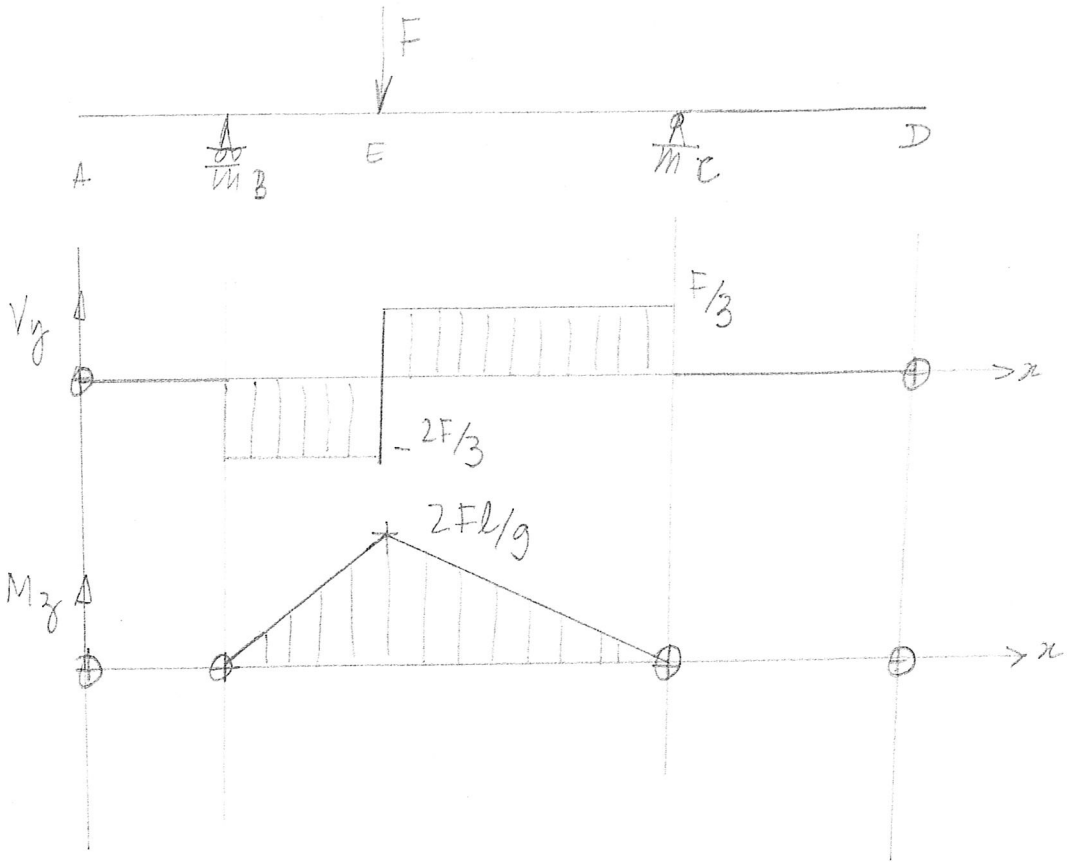
$\boxed{V_{y_{EC}} = \frac{F}{3}}$

$-M_z + Y_C \times (l - x) = 0$

$\boxed{M_{z_{EC}}(x) = -\frac{F}{3} x + \frac{Fl}{3}}$

3°) $V_{y_{BE}}(0) = V_{y_{BE}}\left(\frac{l}{3}\right) = -\frac{2F}{3}$ $\quad M_{z_{BE}}(0) = 0$ $\quad M_{z_{BE}}\left(\frac{l}{3}\right) = \frac{2Fl}{9}$

$V_{y_{EC}}\left(\frac{l}{3}\right) = V_{y_{EC}}(l) = \frac{F}{3}$ $\quad M_{z_{EC}}\left(\frac{l}{3}\right) = \frac{2Fl}{9}$ $\quad M_{z_{EC}}(l) = 0$.



$$M_{zE} = -\frac{3ql}{32}\left(\frac{l}{4}+\frac{l}{3}\right) - \frac{ql^2}{128} + \frac{2Fl}{9}$$

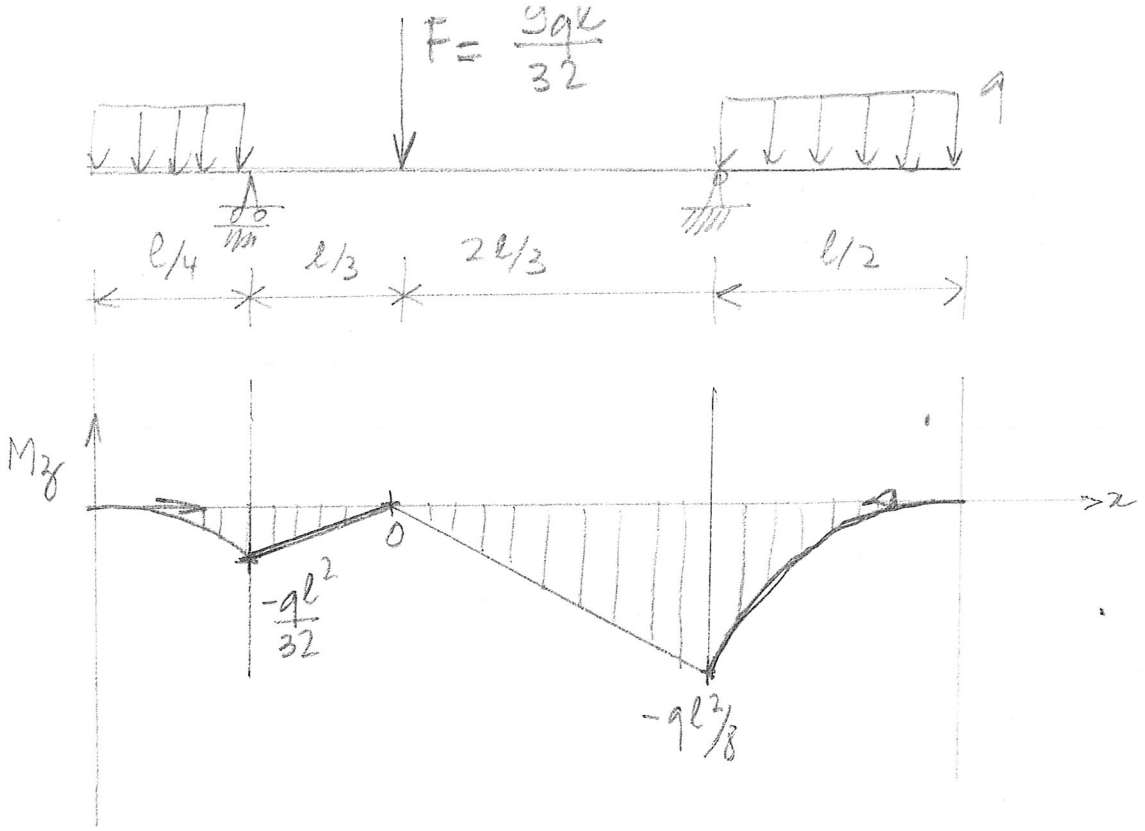
$$M_{zE} = -\frac{3ql}{32} \times \frac{7l}{12} - \frac{ql^2}{128} + \frac{2Fl}{9}$$

$$\boxed{M_{zE} = -\frac{ql^2}{16} + \frac{2Fl}{9}}$$

$$M_{zE} = 0$$

$$\Longleftrightarrow \frac{2Fl}{9} - \frac{ql^2}{16} = 0 \Longleftrightarrow F = \frac{9ql}{32}$$

$$\Longleftrightarrow \boxed{F = \frac{9ql}{32}}$$



AN

$$M_{zB} = -\frac{ql^2}{32} = -\frac{250 \times 6^2}{32} = -281{,}25 \text{ daN.m}$$

$$M_{zE} = -\frac{ql^2}{8} = -\frac{250 \times 6^2}{8} = -1125 \text{ daN/m}$$